Digital Technology as a Tool for Democratizing Powerful Mathematical Ideas*

La tecnología digital como herramienta para la democratización de ideas matemáticas poderosas

A tecnologia digital como ferramenta para a democratização de ideias matemática poderosas

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Abstract

In this article we analyze the role of technology as a means of accessing powerful mathematical ideas (Skovsmose & Valero, 2009) in mathematics education for young people and adults. More particularly, we seek to answer: Can technology act as a bridge between cultural mathematical practices and modeling in favor of a democratic access to powerful mathematical ideas? The possibility of technology being used as a democratizing tool for these ideas is proposed, provided that educational needs are addressed according to previous mathematical conceptions established by the students themselves. In this paper, we present a didactic development, based on the theory of didactictal situations (Brousseau, 1997), as well as the decisions that led to the development of an ad hoc technological tool, serving as a bridge to powerful mathematical ideas. We show how both designs promoted the construction of powerful mathematical ideas and concepts while discussing the possibility of using technology as a democratizing tool of knowledge.

Keywords

adult education; powerful mathematical ideas; digital technology; didactic design; technological design

Resumen

En este artículo se analiza el papel de la tecnología como medio de acceso a ideas matemáticas poderosas (Skovsmose y Valero, 2009) en la educación matemática para jóvenes y adultos. En particular, se busca responder a la pregunta: ¿puede la tecnología fungir como puente entre las prácticas matemáticas culturales y su modelización en favor de un acceso democrático a ideas matemáticas poderosas? Se plantea la posibilidad de que la tecnología se utilice como herramienta democratizadora de dichas ideas, siempre y cuando se aborden necesidades educativas con base en concepciones matemáticas previas establecidas por los propios educandos. En el escrito se presenta un desarrollo didáctico, basado en la teoría de situaciones didácticas (Brousseau, 1997), así como las decisiones que condujeron al desarrollo de una herramienta tecnológica ad hoc, que sirviera como puente hacia ideas matemáticas poderosas. Se muestra cómo ambos diseños promovieron la construcción de conceptos e ideas matemáticas poderosas al tiempo que se discute sobre la posibilidad de usar la tecnología como herramienta democratizadora del conocimiento.

Palabras clave

educación de adultos; ideas matemáticas poderosas; tecnología digital; diseño didáctico; diseño tecnológico

Resumo

Neste artigo é analisado o papel da tecnologia como meio de acesso a ideias matemáticas poderosas (Skovsmose & Valero, 2009) na educação matemática para jovens e adultos. O trabalho tenta responder à pergunta: pode a tecnologia funcionar como ponte entre as práticas matemáticas culturais e a sua modelização para um acesso democrático às ideias matemáticas poderosas? Pondera-se a possibilidade que a tecnologia seja utilizada como ferramenta democratizadora dessas ideias, sempre que sejam abordadas as necessidades educativas com base nas concepções matemáticas prévias estabelecidas pelos educandos. No escrito, apresenta-se um desenvolvimento didático, baseado na teoria de situações didáticas (Brousseau, 1997), assim como as decisões que encorajaram o desenvolvimento de uma ferramenta tecnológica ad hoc, que funciona como ponte para as ideias matemáticas poderosas. Apresenta-se como os dois projetos promoveram a construção de concepções e ideias matemáticas poderosas. Ao mesmo tempo, se debate sobre a possibilidade de utilizar a tecnologia como ferramenta democratizadora do conhecimento.

Palavras-chave

educação de adultos; ideias matemáticas poderosas; tecnologia digital; desenho didático; desenho tecnológico

Introduction

Mexico is one of the countries with the greatest social and economic contrasts in the world. It is enough to walk carefully down the street and recognize in its nooks the coexistence of a minority that enjoys decent housing, first-world health services, and ample educational opportunities, with a large majority living in poverty and under conditions of scarcity. The contexts of life of a minority contrast with a saturated health system, unsafe transportation, overcrowded schools, and early entrance to the job market. This scenario prevails in Latin America.

Inequality translates into large differences in life opportunities and general well-being that not only originate in the dissimilar distribution of economic resources, but also knowledge. There is a directionality, a flow of knowledge and information that is only available to those who have a certain social, cultural, and economic capital. In general, current social structures make this flow inaccessible to our society's low socio-economic strata because, as Skovsmose and Valero (2009) indicate, "all those excluded belong to structurally irrelevant areas in the informational society" (p. 28). According to Flecha (1999):

> [...] knowledge prioritized as a result of new lifestyles is distributed unevenly among individuals, according to the social group, gender, ethnicity, and age. At the same time, knowledge possessed by marginalized groups is not considered, even if it is richer and more complex than prioritized knowledge. More is given to those who have more and less to those who have less, closing a cycle of cultural inequality." (p. 67)

There are many youth and adults who do not have access to these knowledge flows due to the fact that they do not find room within a school culture. Inequalities in access to knowledge and information represent opportunities to rethink everyday life and the possibility of transforming our reality. Education with youth and adults should be part of an effort to attenuate inequalities in access to educational opportunities in pursuit of a society that favors democratizing knowledge; any kind and in every sense, considering what people know and want to know.

How can we face the access to knowledge inequalities? One possible way is by broadening the perspective toward different forms of knowledge, for example, the mathematical, recognizing that based on mathematical social practices, methods emerge that can be revisited to build new mathematical processes. In addition, it is necessary to find means that give more importance to the action over the mere exchange of information. Based on the foregoing, a proposal emerges to inquire into the role of technology as a means to access mathematical ideas that considers what adults know and what they want to know.

Initially, we state the importance of revealing the mathematical aspect in the education of youth and adults by analyzing the concept of *numeralism*. Subsequently, powerful mathematical ideas are presented as a concept that makes the democratic relationships in mathematical education visible. Positing thus the overview, the role of technology is analyzed as a means to access said powerful mathematical ideas, their benefits, and challenges. Finally, we show a didactical and technological design that addresses adult's geometric educational needs, as a practical experience of democratizing mathematical knowledge.

About the Importance of *Numeralism* in Youth and Adults

The education of youth and adults (EDYA) with low-schooling is an understudied educational research line. For almost two decades, regarding the state of adult education, Schmelkes and Kalman (1996) state that:

> Although it is always said that literacy addresses reading and writing skills and basic calculation, the latter is given less importance, although, among adults, it is perhaps deemed like a greater need than learning to read and write. The sense of impotence when faced with deception when completing business operations or labor agreements is a reason frequently indicated by illiterate adults when referring to their need to learn basic calculus. (p. 22)

As of the study cited, some progress has been made in mathematics education with adults with middle and middle-high educational levels, but research work on mathematical education of youth and adults (MEDYA) of low or no schooling has not yet been consolidated. Rather, what is observed is a tendency to investigate teaching and learning processes in mathematical education for children, substantially reducing the number of research dealing with educational processes outside of school or with populations that have had little access to it. This work seeks to contribute to the discussion about MEDYA based on a qualitative work whose approach focuses on low-schooled adults and the possibilities that digital technology offers in this educational context.

About MEDYA

Mathematics, as part of science and knowledge, is a cultural element that allows participating in social and political contexts, for example, in work and school activities. In an "informational society" (Castells, 1996), both science and technology play an active role, considering that "the impact of technology goes beyond industrial production, and, in fact, affects political, economic, social, and cultural structures" (Skovsmose & Valero, 2009, p. 26); thus we see that mathematics is not a neutral element in said structures.

To understand these social phenomena in mathematical education, it is necessary to broaden the perspective about what it means to investigate mathematical education and include the relationship within a broad spectrum that encompasses the extracurricular. Therefore, with the concern of addressing MEDYA so that it includes social, cultural, and political aspects, the term *numeralism* or *mathematical literacy* emerges[1].

This study, recognizing the exercises of power in mathematical activities, *numeralism* is considered as a process that includes visible and invisible actions, within a cultural activity that allows positioning oneself differently in the world. These building processes of a mathematical culture involve appropriating not only a repertoire of concepts, but also *powerful mathematical ideas* (Skovsmose & Valero, 2009) that allow people to visualize themselves and participate differently in everyday situations: the "being more " to which Freire 1970 refers. This term can be extended with the definition of *mathematical literacy*:

> […] The critical role of mathematics in shaping the social world, a citizen's critical competence requires a certain degree of mathematical competence to be able to distance themselves and judge rulers' decisions when those decisions are based on mathematical systems and expert mathematical models. Democratic mathematical competence is not only knowing mathematics to have a basic series of knowledge required in today's labor market, but is also putting that mathematical knowledge into practice to question authorities and, therefore, be able to fight injustice. (Valero, Andrade-Molina, & Montecino, 2015, p. 291).

Reflecting on these *power* processes, together with their own design of activities and the use of mathematical concepts, will favor access to powerful mathematical ideas, provided there is an intentional educational

1 Some authors have described it as numeracy (for example, Street, Baker,& Tomlin, 2008), matheracy (D'Ambrossio, 1996) o mathemacy (Skosmose, 1999).

moment that includes students' social practices as a starting point and a medium (didactical and technological, for example) that organizes moving toward said ideas.

Thus, this paper summarizes a study which began with an inquiry into what adults want to know, what they know, and not based on what they presumably ignore. This work aims to address an important part of MEDYA: not the estimable or utilitarian MEDYA, but rather a builder of bridges between mathematical cultural practices and powerful mathematical ideas, having digital technology as a medium/mediator and a didactical situation.

Based on the definition of the term *numeralism* and the understanding of the educational phenomenon related to MEDYA, below I discuss the importance of considering powerful mathematical ideas as unobservable actions that can be accessed.

About Powerful Mathematical Ideas

Skovsmose and Valero (2009) indicate the existence of powerful mathematical ideas behind observable actions. This notion arises from the need to make power exercises visible that allow participating in certain sociocultural activities when using mathematics. To acknowledge the nature of *power* in these ideas regarding social, cultural, and economic oppression, it is useful to remember that mathematics is constantly used to exclude individuals who need it in their jobs; thus, "what is powerful means asserting the capability of exerting power" (p. 36). In other words, the individual who possesses said mathematical ideas can exert power over other people. Therefore, the idea of *democratizing access to powerful mathematical ideas* arises, understood as "the possibility of entering into a type of mathematical education that favors consolidating democratic social relationships" (p. 48) with the purpose of building, together with the student, possibilities of visualizing action opportunities regarding inequality phenomena. There are four perspectives to understand powerful mathematical ideas:

» From a logical point of view, mathematical ideas linked to *abstraction* provide new ways of understanding the world within an existing set of different concepts, "an intrinsic power within mathematics" (Skovsmose & Valero, 2009, p. 37).

» From a psychological point of view, power is defined with regards to learning potentials. In this sense, Rojano's (2002) definitions of powerful mathematical ideas are the ones that favor access to more abstract, formal, and complex thinking. These ideas are based on a notion of moving between levels of cognition from a lower to a higher level of abstraction. According to this author, some of these notions of movement are:

- From arithmetic to algebra. For example, the [=] sign in arithmetic refers to an action that transforms the left side into what is written on the right side; however, in algebra, the same sign can represent equivalence, equality, or a relationship, or a function.
- From specific to general. For example, when forecasts are done for the number of points in a sequence of them (for example, working in spreadsheets).
- From informal to formal. For example, the idea of progressively refining and systematizing problem solving through trial and error.
- From drawing to geometric figure. It resembles the idea of going from perceptual to conceptual.
- In general, toward a more abstract thought.

Rojano develops an extensive analysis of studies with teenagers, in which cognitive processes are analyzed to specify powerful mathematical ideas as those that enable transition processes between the concrete and the abstract.

» Cultural point of view: "Related to the opportunities students have to participate in a smaller community's or wider society's practices" (Skovsmose & Valero, 2009, p. 43). They are networks of relationships and socially constructed meanings that enabling them to get involved in society in certain cultural actions.

» The sociological point of view. It is related to the scope of certain mathematical resources to be able to act in society. For example, the use of mathematics as descriptive or hypothesis creation tools. This powerful mathematical idea includes a view of the world where there is use of tools (Skovsmose & Valero, 2009, pp. 36-48)

Kaput (1994) considered that democratizing mathematical knowledge could begin at an early age, for example, bringing estimations closer to children. This challenged the idea that advanced mathematics is reserved for a population that reaches higher education and certain careers. For Kaput, reducing the age at which people can access powerful mathematical ideas is a way of democratizing knowledge. In this study, democratizing knowledge is not based on early access to mathematical ideas, but rather on promoting access to powerful mathematical ideas for a wide sectors of society, such as individuals with low-schooling, a sector that has traditionally been excluded from them.

About the Role of Technology

Considering the above scenario, what role can technology play as a tool to access powerful mathematical ideas?

Access

First, to answer this question, it is necessary to acknowledge that the role of technology cannot be contingent to the availability of materials, but rather involves analyzing the processes of use that give meaning to the activities of adults in educational processes and their possibilities of participating in everyday life activities. It is common to use the term digital technologies (DT) to differentiate this position from the one underlying the term Information and Communication Technologies (ICT). This term, ICT , comes from post-industrial economic models. Warschauer (2003) refers to this stage as the "Third Industrial Revolution" (p.13, whereas Castells (2000) refers to it as "informationalism" (p.155). Regarding this "revolution", Herbert Simon (1977) says:

> We are in the first years of a revolution in information processing that shows signs of being as fundamental as the previous energy revolution. We could likely refer to it as the 'Third Information Revolution' (The first produced by written language and the second by the printed book). This third revolution, which began more than a century ago, includes the computer, among other things. Information technology covers a wide range of information storage processes, to copy or transmit it. (p. 1186)

Castells (2000) and Warschauer and Simon (2997) notice within this emergence of new tools, a prevailing trend toward information as a cult. This "informationalist" approach remained until 1994, when the term "new economy" (Laursen, 2004) was coined, subsequently formalized and institutionalized by the Organization for Economic Cooperation and Development (OECD) in 1998. Thus, the term ICT reflects the idea that technology should be centered on the collection and storage of data, rather than taking advantage of it for the common well-being. In contrast to this information cult, the term DT[2] arises to consider that the approach of the technologies lies in the relationship between them and how individuals use them, and not in their "informative" or "communicative" potential.

2 The term DT is used in most of the research within the field of educational mathematics. (See, for example, the minutes of the XVII Meeting of the International Commission on Mathematical Instruction [ICMI] on Technology [Hoyles & Lagrange, 2010]).

This article argues that, on their own, DTs do not enable a change in education beyond certain formal aspects (the automation of problems and answers, the storage and circulation of materials). Instead, the actors are who can transform education, taking advantage of its innovative properties, provided there are adequate (technological) designs and their educational needs are met.

As with the term DT, the notion of *access* is decisive in this research's proposal, since it allows building mathematical action strategies in social situations using technology as a means which transforms behaviors, questions them, allows reflecting on them, and ultimately, modifies them.

Based on socio-cultural theories and redefining the term *access* for written language practices, Kalman (2004) proposes it "refers to the opportunities to participate in written language events where there are situations in which the subject positions themselves ahead of other readers and writers" (p. 26). An expanded use of this term may include DTs and, consequently, make "the transformative practices with which people make identifiable meanings through the use of digital technologies" visible (Gillen & Barton, 2010, p. 9).

A premise to encourage this access is to take, as a starting point, what adults know instead of what we assume they ignore, to subsequently use technology as a bridge toward other ways of representing and accessing powerful mathematical ideas and, subsequently, to opportunities to participate in cultural events. This vision of DTs as a means opposes determinism –where it is assumed that, on their own, these tools directly impact subject's educational development– and shifts the attention to the study of technology as a tool that organizes the process of access to powerful mathematical ideas with the purpose of meeting their educational needs.

One of the purposes of the aforementioned research, was to expose the possibility of carrying out a mathematical educational action that included the use of technologies, which were not so dissociated from cultural mathematical activities, but, on the contrary, resumes them to the greatest extent possible, using the dynamic potential of DTs. The interest lies in analyzing the role that *technologies* can play in MEDYA, revisiting students' previous mathematical conceptions, based on their social mathematical practices and linking them with powerful mathematical ideas.

Benefits Evidenced from Using DTs in Mathematics Education

The ways to express mathematical knowledge vary depending on the medium of representation used. For example, using a pencil and paper allows certain actions that DTs do not and vice versa. Representing mathematical knowledge in different ways emphasizes and facilitates working with different sections of concepts. For example, in a spreadsheet, by noticing the dynamic changes between a data table and its graphical representation, we can highlight the slope of the graph and formulate hypotheses about how the graph's inclination changes depending on the value of *m* in . Another example is the representation of geometry that dynamic geometry programs show, which have changed how to interact with contents.

Together with a didactical and suitable software design, DT could open up the possibility to accessing mathematics intrinsic to said representations and, consequently, learn by interacting with mathematical objects. Specifically, different ways of representing mathematical knowledge favor (or not) working a mathematical concept in a certain way; the concept's representation and the work method emerge simultaneously.

Focusing on mathematics education, the plurality of technological uses depends on the designs and, most importantly, on the practices around these technologies. Some uses of DT establish new ways of linking with ideas or concepts, especially those in which mathematical activity changes how students traditionally relate to knowledge, which generates new links between human activity and representations modeled by technology. DTs, their virtual environments, uses, representations, and tools can collaborate in reflecting powerful mathematical ideas at both macro (institutional and social) and micro (didactical and cognitive) levels.

Below, evidence of the benefits derived from the use of DTs in mathematics education is presented. One of them, is to receive instant feedback when interacting with mathematical objects, which encourages students to conjecture and continue exploring (Clements, 2000), such as when the Logo program is run and debugged (Nastassi, Clements, & Battista, 1990). Also, it is evidenced that using technology to draw geometric figures promotes experimentation processes and the formulation of conjectures (Jarret, 1998; Ruthven & Hennessy, 2002); for example, when modifying the vertices of a triangle in a dynamic geometry program and noticing that the sum of the interior angles is constant.

In Latin America, the studies of Butto & Rojano (2004, 2010) and Rojano (2002, 2003), show how introducing technology can give access to ideas and processes of mathematical generalization at an early age, from 10 to 11 years old. Based on these studies, it is possible to show the feasibility of presenting basic education students with algebraic thinking, taking advantage of arithmetic topics and programming language potential. Thinking specifically about numeralism aspects, the study by Geiger, Goos, and Dole (2015) shows how DTs, in the classroom, can improve how mathematical elements are approached, provided they are related to extra-curricular contexts.

These examples, of the benefits from using DTs mostly correspond to what has been studied with school-age children population. A good challenge –and part of what is of interest in this study– is analyzing the possibility that said benefits may impact low-schooled youth and adults. It should be noted that adults hold mathematical conceptions beforehand that can be enhanced with the use of DTs, in addition, they give a social value to the use of said technologies.

Challenges of Using DT in MEDYA

Research on incorporating DT in MEDYA implies analyzing some challenges. Similar to what Rojano (2002) discusses.

> ... it also can be said that the introduction of information technology to the mathematics classroom has brought with it the possibility of democratizing mathematical knowledge, [...] which previously was only accessible to a minority of students wanting to follow a scientific university degree. (Rojano, 2002, p. 158)

I regard a challenge for DTs in MEDYA is precisely opening up the possibility of democratizing knowledge outside of the classroom considering educational processes that function as bridges between what adults know and what they want to know.

If technological designs can achieve adequate cognitive challenges for adults using the potential of technology to be reflected in DTs, it is possible to think about educational events for MEDYA that give rise to democratization processes of mathematical ideas that do not cognitively infantilize activities. Said challenges can be designed by directly manipulating executable objects within DTs, keeping mathematical properties and promoting their use based on a didactical situation.

From a numeralism perspective, in which it is possible to view a process mediated by society in which a mathematical culture is built based on the alignment between cultural actions and mathematics, it is possible to distinguish the latter as social practices and, above all, as cultural action. This point of view concurs with Hoyles and Noss (2003, 1996) with respect to distinguishing mathematics as an activity. Methodologically, "[...] we can introduce new notions and try to understand how the thinker connects these with what he or she already knows" (Noss & Hoyles, 1996, p. 9). Intending low-schooled adults to learn mathematics formal, institutional, and determined by certain algebraic rules– can be substituted for the intent to develop and put into practice mathematical attitudes such as hypothesis creation, experimentation, corroboration, and comparison between ideas; all this based on analyzing the academic needs identified by students.

By conceiving mathematics as an activity, an open field is created for the teaching of modeling processes (and generalization), without having to elaborate abstract constructs or a formal algebraic language "from scratch". Instead, the interactions between what is concrete and abstract are taken as the pillar of the mathematical idea of *modeling*. As will be shown, in corresponding technological application and didactical design, preference is given to the incentive of mathematical attitudes, and thus to the interaction between what is concrete and what is abstract mediated by technology. This interaction constitutes, in itself, the way in which learning[3] is conceived in this research: a learning defined by the modification of knowledge and know-how, subsequent to putting the knowledge into practice. When putting previous conceptions into practice in a cognitive conflict situation, learning occurs and, subsequently, based on the learning experiences, mathematical ideas are constructed that may or may not be significant (depending on the experimental difficulties and the student's characteristics).

It is a conflict situation inspired by Piaget's cognitive conflict, where the student restates their schemes and moves to another structure, questioning the previous one.

The challenge of creating material using DT for MEDYA involves reflecting on the characteristics of a mathematical software. In this regard, Noss and Hoyles (1996) indicate the need for it to become a *window of representations and mathematical processes*. In theory, it has to provide an arena in which the student's representations of mathematical concepts are put into play –in this case, a low-schooled adult– conflicts are created, adequate feedback is provided, and learning occurs[4]: "A software that fails providing the student with the means of expressing mathematical ideas, does not open any window whatsoever to mathematics learning processes" (Noss & Hoyles, 1996, p. 54).

Ideally, the mathematical software changes the common representations of knowledge and presents, manipulates, and develops them in another way (DiSessa, 1988). This difference consists in, semiotically, changing how mathematical knowledge is represented and interacted with. The software designer can deliberately present a mathematical knowledge

3 Based on constructivism, specifically on Brousseau's (2007) definition, according to which "the subject learns by correcting their actions and anticipating their effects" (p. 54).

4 Without excluding the teacher's role.

in such a way so the student builds the notion by acting on it, as indicated by Noss and Hoyles (1996). Specifically, the contribution of this theory of windows into mathematical knowledge (Noss & Hoyles, 1996) is that software development and didactical situations, ideally, compare the representations and organize the movement between conceptions prior to the educational moment and powerful mathematical ideas.

When using computers, adults can contrast the mathematical representations they have used up to that moment with those it displays. Then, technology comes in as a *medium* (in Brousseau's style) that creates possibilities for the student to compare their previous notions and, therefore, modify knowledge and learn. Thus, technology is conceived as a bridge to *powerful mathematical ideas*.

In accordance with the position that DTS on their own do not generate a change, it is important that technological development be accompanied by a didactical situation that organizes it. Didactical sequences try to encourage students to "anticipate" certain problems. These anticipations are different ways of representing and treating knowledge and, therefore, signs of learning. For example, if the adult tends to think that the area of any surface can always be calculated as "base times height", the technology and didactical situation will show that there are figures in which said formula is less precise and, therefore, they will have to modify their thinking and create a thought structure relative to *anticipation*.

One of the challenges of MEDYA is redefining adult educational practices in relation to access to DTS. This opens up the possibility of building a new way of understanding this population's educational processes in terms of social, economic, and cultural aspects. For teachers and researchers, dedicated to mathematics for this population, the challenge is to conceive technology as a useful tool in the educational space for youth's and adults' mathematics teaching and learning processes.

Possible Uses of DTS in MEDYA

The lack of studies on the use of DTS and MEDYA offer a fertile field of research in several lines. The first is that, since MEDYA's educational field has not been not fully identified, the uses of DTS can be reported within other fields (for example, rural education, professional education, migration) and cannot be identified as part of the adult education field nor analyzed jointly. On the other hand, the bridge that DTS can build as access routes to mathematical ideas are based on the democratization of knowledge.

Based on the foregoing, three arguments are posited regarding what DTs contribute to MEDYA.

1. DTs generate new ways to express mathematical knowledge and, therefore, new ways of acting on this knowledge, which is why they open up the possibility of building a bridge between cultural mathematical actions and powerful mathematical ideas, using mediation, in a double task of representation, both of cultural actions as of mathematical models. This conception is of DT as a tool linking what is concrete and what is abstract.
2. When technology is conceived as a cultural mediator between the subject and knowledge, it modifies the relationship students have with mathematical concepts and ideas, and therefore, the didactical task and cognitive action. Consequently, it is possible to create a didactical and technological design that positions the student as an agent in a mathematical culture and allows them to experience first-hand mathematical ideas that, without technology, would only exist in a theoretical dimension.
3. The potential that technology has to open communication channels with concepts and powerful mathematical ideas. In other words, with the mediation of DTs, it is possible to analyze and act on powerful mathematics ideas and, therefore, democratize knowledge.

Below, there is an example of how technology can play the role of a bridge, as a tool that broadens how it is represented and how it interacts with mathematical knowledge and, ultimately, as a tool that enables democratizing powerful mathematical ideas in light of specific educational needs.

Example of a Tool to Access Powerful Mathematical Ideas

This example arises from an investigation that analyzes the role of DTs in the low-schooled MEDYA field. The matter was approached was by developing a didactical design based on the theory of didactical situations (Brousseau, 1997), as well as an ad hoc technological tool that functions as a bridge to powerful mathematical ideas.

Preliminary analysis.

First, a series of interviews were conducted with five adults who have not completed their basic education, documenting their educational needs[5].

5 Although there were other types of educational needs, in this research, I will only discuss geometric needs.

Adults' Geometric Educational Needs.

The topic of calculating areas arose from the request of several adults that were experiencing difficulties in calculating areas that were not rectangles. For example:

> Interviewer: [...] If you need, for example, to measure a wall, uh, you said to flatten it, you calculated the square meters squared, right? Is there a difference in calculating square meters of a wall, or of a ceiling, or of a floor, or is it always the same?
>
> Claudia: Well, it is the same, in other words, to measure the ceiling or the wall. Well, I measure it length by width. (Personal communication,05, 03,2014

Just like Claudia, all other participants calculated the area of any surface by multiplying length by width, regardless whether it was a triangle, circle, or whatever it was. The remainder was a "natural" part of the calculation and was considered with the loss or surplus of material.

In the following extract, it is possible to see how one of the women interviewed, has the need of understanding how to calculate the money charged for flattening a floor:

> Yeni: Well, my husband is a blacksmith and he, just by looking, he worked it out. And I am like: where or how did you see it? Explain to me why? The same with the cubic meters of houses. I saw with one of my uncles, who is a bricklayer, and I wonder, how does he get that answer, right?
>
> Interviewer: As in cubing, right?
>
> Yeni: Yes. I hear them saying: 'I charge them per flattened meter or by tile. If it is by tile, I charge, 4 by 4'. They are the only ones that understand what they are talking about. No, the truth, I am clueless. (Personal communication, 06, 03,2014

What Yeni tells us, reflects her interest in knowing certain mathematical notions so she can use them. Learning how areas of houses are measured and, therefore, how much is charged for a job, could develop her participation in certain work activities that, up to that moment, were impossible for her.

Calculating areas also appeared in Felipe's request:

> Interviewer: But, for example, in your everyday life, in your work, have you required any mathematics that you were unable to do?
>
> Felipe: Yes […] Well, here, in my work, I once needed to know the volume of water stored in the tank. Then, I did not know how to calcu-

> late cubic meters and, of course, I was a little frustrated. I had to ask a mathematics and circuits teacher. I did not know how much water was needed to fill a tank. So, he more or less told me to: 'multiply this by that to obtain the cubic meters'. And that was how the problem was solved. Obviously, I did not tell my supervisor: I do not know. But, I was embarrassed, I was frustrated and told him: 'I will tell you in a minute". Then, I resorted to the teacher and afterwards ... well it is 80 cubic meters. To what my supervisor answered: 'oh, I am glad that you know how to calculate it.' He did not know that I had asked for help.

The educational need was documented during the interview when Felipe showed us that his inability to calculate the volume of water needed to fill a tank, made him feel frustrated. Once again, there is a need to know how to calculate areas and volumes to be able to participate in a work activity. Other educational needs found during the interviews were: 1) calculate the area of some wall to charge per square meter flattened and 2) calculate the area of a certain plot of land and request a certain amount of fertilizer or cement.

Faced with this situation, an alternative to traditional education was proposed, where a formula is taught for each geometric figure (a formula for a square, another for a triangle, etc.). In contrast, for this research, we chose to explore the use of Pick's theorem as the single formula with which to calculate the area of polygons with integer vertices.

Pick's theorem states:

> For a simple polygon, whose vertices have integer coordinates, if *B* is a number of integer points on the edge and *I* the number of integer points inside the polygon, then the area *A* of the polygon can be calculated with the formula:

$$A = I + \frac{B}{2} - 1$$

Traditionally, a school curriculum (for example, in Mexico) proposes the triangulation method to calculate the area of irregular figures. The issue is that, in many cases, the Pythagorean theorem or trigonometry are needed to calculate the area of the triangles and they are only taught after the third year of secondary school; in other words, individuals without a basic education would be unable to apply this knowledge. In contrast, Pick's theorem changes the cognitive approach to calculating areas by using basic arithmetic and counting as the basis for calculating an area, opening the possibility to calculating irregular figures. We thought of this theorem because of the following benefits for the specific population with which we work:

» Use and systematization of adults' previous mathematical conceptions.
» Access to calculating the areas of a large repertoire of irregular figures.
» Little memorization of formulas.

Thus, the theorem emerges as an option that comes from the institutional mathematical world; however, being aware of the challenge posed by using the theorem in isolation, an *ad hoc* technological design and a didactical sequence were created to organize educational moments with students.

Technological design.

An *area calculator* consists of a grid in which vertices can be placed until closing a figure. By doing so, the interior and perimeter points appear with integer coordinates (see Figure 1).

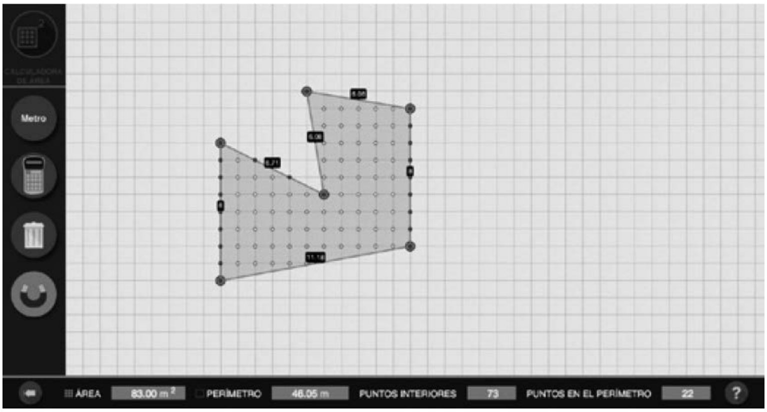

Figure 1. Surfaces calculator

In a Didactically, the need of creating a digital environment arises from the graphic representation promoted by the theorem. For example, observing in real-time where and how the interior points and those on the border are placed when closing the figure. With this *ad hoc* tool, it is possible to edit the position of the vertices and recognize thus how, based on these changes, the area or perimeter changes dynamically.

Didactical design.

The didactical sequence begins with a "circle of culture" (Freire, 1970, p.129 that encourages reflecting on different social issues that problematize putting technology into practice in education. These topics are extremely sensitive for the adults we worked with, but three examples can be: 1) problematizing the difference between children and adult education; 2) technology as a man-made creation for human beings, or; 3) reflecting on meaningful learning compared to learning by isolated concepts. These topics encompass the entire didactical sequence and give MEDYA the social meaning it deserves.

Subsequent to the dialogue, we present a *problem generator*, which asks to calculate the area of the following figure however possible:

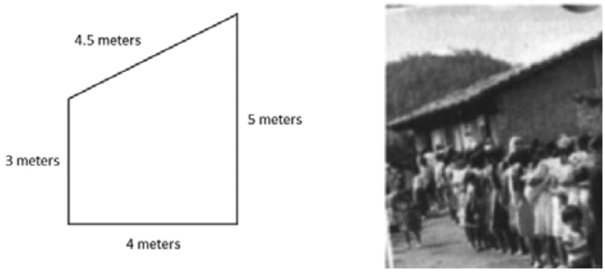

Figure 2. Generated problem

This problem is based on two academic backgrounds: Agüero (2006) and Estrada and Ávila (2009), who proposed it as a problem to calculate an area fwith or low-schooleding adults. The following didactical steps are as follows:

» Written and oral presentation of Pick's theorem.
» Using the theorem and *ad hoc* technological application with rectangles that serve to corroborate the previous conception of $b \; x \; h$.
» Determining the triangle formula by experimenting on the technological application.
» Calculating areas of regular figures.
» Calculating areas of irregular figures.
» Estimating as a process to calculate problems in everyday life.
» Contextualized problems.
» Returning to the problem generator.

Each of these steps refers to the theoretical elements of the theory of didactical situations (Brousseau, 1997), such as institutionalization, validation, devolution, and dialectical formulation. This didactical sequence covers, approximately, three sessions of two hours each.

Technology as a bridge to powerful mathematical ideas.

When using the technological tool together with the didactical sequence, the following results were documented regarding technology viewed as a bridge to powerful mathematical ideas:

From the drawing to the geometric figure.

We recorded that, the study of notions, such as area and perimeter, requires building a bridge between the drawing and the geometric figure. This occurred during the didactical sequence when analyzing walls or plots to be subsequently outlined in a geometric figure. Something interesting is that the students themselves acknowledged that when modeling the plots, a precise calculation could not be obtained because of small errors intrinsic to gauging. Also, something noteworthy is that, when analyzing a geometric figure that can be embedded in a grid, the importance of analyzing how to distinguish whether it is or is not a lattice point arises, in other words, if it has integer coordinates. This fact suggests that, in an evolving didactical sequence, the next step is to analyze how to decide on whether a point on a line has integer coordinates given a segment with integer vertices.

From the specific calculation method to general methods.

When conducting the preliminary studies, we recorded that the adults built ways to calculate the area of irregular figures using the knowledge of $b\ x\ h$. Although with this knowledge there is a broad repertoire of figures, there can be major errors in those that do not resemble rectangles.

By accepting the idea that it is possible to calculate the area of any polygon whose vertices can be embedded in a lattice by using Pick's theorem refines the estimate, turning the knowledge of $b\ x\ h$ into one more method to choose from among a collection of the adults' previous mathematical conceptions. Access to this knowledge was achieved by developing a virtual lesson comprised of a didactical situation and the *ad hoc* development. Choosing Pick's theorem functioned as a bridge between specific calculation methods and general methods, in large part because of the analysis of previous conceptions (especially the previous calculation methods found when exploring previous mathematical conceptions); and, in particular, the basic arithmetic used by the theorem, the simple expression in an equation and its graphic expression in the *ad hoc* development.

In general, the cognitive challenges that students faced with the tool and the didactical sequence were regarding a more abstract thinking with which to calculate the area of a greater number of figures. By abstracting the idea that the figure exists within a lattice, it is possible to count its interior and border points based on its sides' measurements. In addition to this, we recorded the construction of the meaning of terms, such as area and perimeter.

Conclusions

The didactical sequence, together with the technological tool, enabled students to not only build and use new concepts, but also powerful mathematical ideas. Said knowledge building ensued from not infantilizing cognitive challenges posed by the sequence, for example, using an algebraic formula to present a theorem to low-schooled adults (several of them without secondary education).

Adults have enough cognitive skills and interest to deal with concepts and representations worked on during the didactical sequence. The theoretical position taken in this research posits the starting point of an educational process with low-schooled adults based on what they do know rather than what we assume they ignore.

Faced with the lack of visibility of the educational processes incorporating technology for MEDYA, the results presented in this research position themselves as an opportunity to analyze the potential of DTS by creating an *ad hoc* didactical and technological design specific to this population as a knowledge democratizing tool.

Acknowledging technology as a tool helps to see two important aspects: the social part of introducing a tool that is foreign to everyday life and the expeditious use given to said instruments. Conceiving technology as a means to access knowledge will enable characterizing its didactical possibilities, for example, planning how the student will face their previous mathematical knowledge and analyzing how feedback from the means to the student could possibly work.

This research shows that technology offers the possibility of democratic access to ideas that were usually limited to people in school processes. In this case, adults have access to certain powerful mathematical ideas through the development of an *ad hoc* program and the didactical sequence constructed based on their educational needs. That technology functions as a democratizing tool is favored both by the *ad hoc* design, in other words, how mathematical objects can be manipulated, and by the didactical design organizing the situation, but, above all, by how adults relate to said technology, which meets their educational needs.

References

Clements, D. (2000). From exercises and tasks to problems and project - Unique contributions of computers to innovative mathematics education. *The Journal of Mathematical Behavior, 19*(1), 9-47.

Camargo, L. & Sandoval, I. (2017). Acceso equitativo al razonamiento científico mediante la tecnología [Equitable access to scientific reasoning through technology]. *Revista Colombiana de Educación,* (73), 179-211.

Castells, M. (1996). *La era de la información. Economía, sociedad y cultura [The information age: Economy, Society and Culture]*. Ciudad de México, México: Siglo XXI.

Castells, M. (2000). *The rise of the network society.* Blackwell: 2d ed. Malden, Mass.

Laursen, K. (2004). New and old economy: the role of ICT in structural change and economic dynamics. *Structural Change and Economic Dynamics Special issues.*, 241-243.

Agüero, M. (2006). *El pensamiento práctico de una cuadrilla de pintores estrategias para la solución de problemas en situaciones matematizables de la vida cotidiana [The practical thinking of a painting crew: Strategies for solving mathematical situations in daily life]*. Pátzcuaro, Michoacán, México: CREFAL- Universidad Iberoamericana.

Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. (M. C. Nicolas Balacheff, Ed.) Dordrecht, Netherlands: Kluwer Academic Publishers.

Brousseau, G. (2007). *Iniciación al estudio de la teoría de las situaciones didácticas [Introduction to study the theory of didactic situations].* (D. Fregona, Trans.) Buenos Aires, Argentina: Libros del Zorzal.

Butto, C., & Rojano, T. (2004, April). Introducción temprana al pensamiento algebraico: abordaje basado en la geometría [Early introduction of algebraic thinking: Geometry bases approach] . *Educación Matemática, 16*(1), 113-148.

Butto, C., & Rojano, T. (2010, April). Pensamiento algebraico temprano: El papel del entorno Logo [Early algebraic thinking: The role of the environment Logo]. *Educación Matemática, 22*(3), 55-86.

D'Ambrosio, U. (1996). *Educacão matemática: Da teoria à pràtica [Mathematics education: From theory to practice].* Campinas, Brasil: Papirus Editora.

diSessa, A. (1988). Social Niches for Future Software. In A. diSessa, M. Gardener, J. Greeno, F. Reif, A. Schoenfeld, & E. Stage, *Towards a Scientific Practice of Science Education.* New Jersey, NY, USA: Lawrence Erlbaum associates.

Estrada, J. L. & Ávila, A. (2009, December). Los usuarios de la educación básica para jóvenes y adultos y la solución de un problema de área [Users of basiceducation for Youth and Adults and the solution of an area problem]. *Educación Matemática, 21*(3), 33-66.

Flecha, R. (1999). New Educational Inequalities. In M. Castells, *Critical Education in the New Information Age* (pp. 65-82). Cumnor HIll, Oxford, England: Rowman & Littlefield Publishers, INC.

Freire, P. (1970). *Pedagogía del Oprimido [Pedagogy of the Oppressed]* (30a Edición.). México: Siglo Veintuno Editores.

Geiger, V., Goos, M., & Dole, S. (2015). The role of digital technologies in numeracy teaching and learning. (N. S. Council, Ed.) *International Journal of Science and Mathematics Education*, 1115-1137.

Gillen, J. & Barton, D. (2010). *Digital Literacies. A Research Briefing by the Technology Enhanced Learning phase of the Teaching and Learning Research Programme.* London: London Knowledge Lab, University of London.

Guerrero, I. (2012, January-April). De la brecha a los múltiples caminos: posibilidades de la tecnología en educación [From the breach to multiple paths: Possibilities for Educational Technology]. *Decisio*(31), 3-8.

Hoyles, C. & Lagrange, J.-B. (2010). *Mathematics Education and Technology-Rethinking the Terrain.* New York, Dordrecht, Heidelberg, London: Springer.

Hoyles, C. & Noss, R. (2003). What can digital technologies take from and bring to research in mathematics education? *Second international handbook of mathematics education*, 323-349.

Huerta, M. G. (2012, January-April). El tiempo en que los jóvenes y adultos se apoderaron de la tecnología para la vida y el trabajo [The time when youth and adults took ownership of technology for life and work]. *Decisio*, 15-19.

Jarret, D. (1998). *Integrating technology into middle school mathematics. It's just good teaching.* Northwest Regional Educational Laboratory.

Kaput, J. J. (1994). Democratizing access to calculus: New routes using old roots. (A. Schoenfeld, Ed.) *Mathematical Thinking and Problem Solving*, 77-155.

Nastassi, B. K., Clements, D. H., & Battista, M. T. (1990). Social cognitive interactions, motivation, and cognitive growth in Logo programming and CAI problem-solving environments. Journal of Educational Psychology, 82, 150±158. *Journal of Educational Psychology, 82*, 150-158.

Noss, R. & Hoyles, C. (1996). *Windows on Mathematical Meanings: learning cultures and computers*. Netherlands: Mathematical Education Library. Kluwe Academic Publishers.

Rojano, T. (2002). Mathematics Learning in the junior Secondary School: Students' Access to Significant Mathematical Ideas. (L. D. English, Ed.) *Handbook of International Research in Mathematics Education*, 143-164.

Rojano, T. (2003). Incorporación de Entornos Tecnológicos de Aprendizaje a la Cultura Escolar: Proyectos de Innovación Educativa en Matemáticas y Ciencias en Escuelas Secundarias Públicas en México. *Revista Iberoamericana de Educación.*, 135-169.

Ruthven, K. & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics, 49*(1), 47-88.

Schmelkes, S. & Kalman, J. (1996). *La educación de adultos: estado del arte. Hacia una estrategia alfabetizadora para México [Adult education: State of the art. Towards a literacy strategy for Mexico].* (INEA, Ed.) México City: Instituto Nacional para la Educación de los Adultos.

Simon, H. (1977, 03 18). What computers mean for man and society. *Science, 195*, 1186-1991.

Skovmose, O. (1999). *Hacia una filosofía de la educación matemática crítica [Towards a philosophy of critical mathematics education].* (P. Valero, Trans.) Bogotá: Universidad de los Andes.

Skovsmose, O., & Valero, P. (2009). Democratic access to powerful mathematical Ideas. In L. D. English, *Handbook of international research in mathematics education.* (pp. 383-408). New York: Routledge.

Street, B., Baker, D., & Tomlin, A. (2008). *Navigating Numeracies: Home/School Numeracy Practices.* London: Springer - Science.

Valero, P., Andrade-Molina, M., & Montecino, A. (2015). *Lo político en la educación matemática: de la educación matemática crítica a la política cultural de la educación matemática [The politics in mathematical educaction: from critical mathematics education to cultural politics of mathematics education.* (Vol. 18). Mexico City: Colegio Mexicano de Matemática Educativa A.C.

Warschauer, M. (2003). *Technology and social inclusion: rethinking the digital divide.* Massachusetts: Massachusetts Institute of Technology.